Grade 8 Mathematics, Quarter 3, Unit 3.1

Solving One-Variable Equations Transitioning into Two Variables

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve linear equations with approximation and exact reasoning methods.
- Use equations to represent questions about problem situations and interpret the solutions in the context of the problem.
- Explore relationships between paired values of numerical attributes.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Monitor and evaluate progress and change course if necessary.
- Check answers to problems using a different method, and continually ask, "Does this make sense?"

Construct viable arguments and critique the reasoning of others.

- Justify conclusions, communicate them to others, and respond to arguments.
- Listen to and read arguments of others and decide whether the arguments make sense, while clarifying and working to improve the argument.
- Apply mathematics to solve problems arising in everyday life, society, and the workplace.

Model with mathematics.

- Maintain oversight of the process while working on a problem and attend to the details.
- Evaluate the reasonableness of intermediate results.

Essential questions

- How is the order of operations utilized in solving an equation?
- Can an equation have more than one solution? How do you know?
- What does the variable represent in the equation that was solved?
- Can you write an equation for a story problem and use it to solve for the given situation?
- How can you determine whether a scatter plot has a positive or negative association?
- What are the outliers of the data represented by the scatter plot?
- What information do you need to locate a point on a scatter plot?

Written Curriculum

Common Core State Standards for Mathematical Content

Expressions and Equations 8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In kindergarten through 2nd grade, students represented and solved problems involving addition and subtraction. In grades 3 and 4, students used the four operations with whole numbers to solve problems. In 3rd grade, students represented data with a line plot. In grade 5, students learned to write and interpret numerical expressions. In 4th and 5th grades, students analyzed patterns and relationships. In 6th grade, students applied reasoning to solve one variable equations and inequalities. Students also represented and analyzed quantitative relationships between dependent and independent variables. In grade 7, students used properties of operations to generate equivalent expressions and used variables to represent quantities in real-world or mathematical problems.

Current Learning

Students solve linear equations in one variable. Students investigate patterns of association in bivariate data.

Future Learning

In high school algebra courses, students will solve quadratic equations with real coefficients that have complex solutions. They will create equations in two or more variables to represent relationships between quantities. Students will explain each step in solving a simple equation and construct a viable argument. Students will solve simple rational and radical equations in one variable, and give examples of showing how extraneous solutions may arise. Students will interpret the correlation coefficient in a linear fit.

Additional Findings

In the beginning, students may find it unnecessary to show their work in solving simple one-step equations. Encourage them to show all steps in isolating the variable in order to develop a process for solving more complex problems. Students may also have trouble understanding that the equation $x + 5 = 7$ is equivalent to $1x + 5 = 7$. Remind them of the identity property ($1 * x = x$) and allow them to insert the one if it puts them at ease. Also, students may have difficulty identifying positive and negative correlations due to reading the scatter plot incorrectly. Emphasize that all graphs should be read from left to right.

Grade 8 Mathematics, Quarter 3, Unit 3.2

Defining, Evaluating, and Comparing Functions

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Use tables, graphs, and equations of linear relationships to answer questions.
- Solve problems and make decisions about linear relationships, using information given in tables, graphs, and symbolic expressions.
- Understand the connection between linear equations and the patterns in the tables and graphs of those equations; use rate of change, slope, and y-intercept.
- Construct tables, graphs, and symbolic equations that express linear relationships.
- Recognize problem situations in which two or more variables have a linear relationship to each other.
- Describe the pattern of change between the independent and dependent variable for linear relationships that are represented in tables, graphs, and equations.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain the correspondence between verbal descriptions, tables, and graphs.
- Transform algebraic expressions or change the viewing windows on graphing calculators to get the information needed.

Construct viable arguments and critique the reasoning of others.

- Analyze situations by breaking them down into cases and recognize and use counterexamples.

Look for and make use of structure.

- Discern a pattern of structure.
- Step back from an overview and shift perspectives.

Essential questions

- Is the relationship represented by a given graph (generated by instructor) linear?
- How would the rate of change be found in a table, graph, and equation?
- Can two inputs have the same output within a function? Explain.
- What information is given from a point of a function that is graphed?
- How do you determine which equation has the greatest rate of change?
- How can you tell if a relationship is linear from a table, graph, and equation?
- How can you write an equation of a line, given a point and slope or two points on the line?
- What is the difference between inverse and linear relationships?

Written Curriculum

Common Core State Standards for Mathematical Content

Functions	**8.F**

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

[1] Function notation is not required in Grade 8.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 5, students analyzed patterns and relationships. In grade 6, students analyzed quantitative relationships between dependent and independent variables. In grade 7, students used properties of operations to generate equivalent expressions.

Current Learning

Students understand that a function is a rule that assigns each input to exactly one output. They interpret the equation $y = mx + b$, defining a linear function whose graph is a straight line, and they give examples of functions that are not linear.

Future Learning

In high school algebra courses, students will solve systems of equations using both linear and quadratic functions. They will create equations and inequalities in one variable and use them to solve problems with coefficients represented by letters. Students will construct and compare linear and exponential models to solve problems. They will build functions that model relationships between two different quantities using different representations.

Additional Findings

It will be beneficial for students to familiarize themselves with the graphing calculator, acquiring skills in changing the window, graphing a line, accessing and setting up the table (table set), calculating a point of intersection, calculating the output of a function given the input, and interpreting the graph obtained.

Grade 8 Mathematics, Quarter 3, Unit 3.3

Using Functions to Model Relationships

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Recognize linear and nonlinear patterns in tables and graphs.
- Describe data patterns using words and symbols.
- Write equations to express patterns appearing in tables, graphs, and word problems.
- Interpret the slope and *y*-intercept in the context of the question.
- Sketch a graph for a given equation, table, or word problem.
- Determine the slope and *y*-intercept given a table, equation, or description of a linear relationship.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Make conjectures about the values within a table and test the table to see whether it works.
- Reason inductively about data.

Model with mathematics.

- Write an equation to model a real-life problem.
- Interpret mathematical results in the context of the situation and reflect on whether the results make sense.

Attend to precision.

- Label axes when graphing to clarify the correspondence with quantities in a problem.

Essential questions

- What information do the variables and numbers represent in an equation?
- What does the *y*-intercept and slope mean in the context of the question?

Written Curriculum

Common Core State Standards for Mathematical Content

Functions	**8.F**

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students analyzed quantitative relationships between dependent and independent variables. In grade 7, students used properties of operations to generate equivalent expressions.

Current Learning

Students understand the connections between proportional relationships of lines and linear equations. They analyze and solve linear equations and pairs of simultaneous linear equations. Students use functions to model relationships between quantities and understand that a function is a rule that assigns each input to exactly one output.

Future Learning

In high school mathematics courses, students will graph functions of linear and quadratic equations. They will continue to solve systems of equations using both linear and quadratic functions. Students will create equations and inequalities in one variable and use them to solve problems with coefficients represented by letters. Students will construct and compare linear and exponential models to solve problems. They will build functions that model relationships between two different quantities using different representations.

Additional Findings

At first students may have trouble finding slope since they will interchange the changes in y and x. Students may also have trouble understanding whether a line is increasing or decreasing. Emphasize reading the graph from left to right in the same manner that one reads a book.